

## VI Semester B.Sc. Examination, September - 2021

## (Semester Scheme) (CBCS)

## MATHEMATICS

## Complex Analysis - II and Improper Integrals (Sec - 03)

Time : 2 Hours

Max. Marks : 40

- Instructions :*
- 1) Answer all questions.
  - 2) First question carries 10 marks and remaining questions carry 15 marks.

1. Answer any five questions. Each question carries two marks.

- a) Find  $\int_C \bar{z} dz$  where C is the line segment joining  $-i$  to  $i$ .
- b) Evaluate  $\int_C (x + 2y)dx + (3y - x) dy$  along the curve  $y = x^2$  from  $(0, 1)$  to  $(1, 2)$ .
- c) Evaluate  $\int_C \frac{dz}{(z-2)^4}$  around the circle  $|z-1| = 5$ .
- d) State Fundamental theorem of algebra.
- e) Evaluate  $\int_0^{\infty} x^5 e^{-x} dx$ .
- f) Prove that  $\Gamma(n+1) = n \Gamma(n)$ .
- g) Evaluate  $\beta(5, 4)$ .
- h) Evaluate  $\int_0^1 x^7 (1-x)^5 dx$ .

P.T.O.

2. Answer any three questions. Each question carry five marks.

- a) Evaluate  $\int_C (x + 2y)dx + (4 - 2x)dy$  around the ellipse  $x = 4 \cos\theta, y = 3 \sin\theta, 0 \leq \theta \leq 2\pi$  where C is taken in anticlockwise direction.
- b) Evaluate  $\int_C \frac{3z-1}{z^3-z} dz$  where C is the circle  $|Z| = \frac{3}{2}$ .
- c) State and prove Cauchy's Integral theorem.
- d) Evaluate  $\int_C |Z|^2 dz$  where C is the rectangle whose vertices are (0, 0), (2, 0), (2, 1) and (0, 1).
- e) State and prove Liouville's theorem.

3. Answer any three questions. Each question carries five marks.

- a) Show that  $\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\cos\theta}} d\theta = \pi$ .
- b) Prove that  $\Gamma(n) \Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}} \Gamma(2n)$ .
- c) Evaluate  $\int_0^1 \frac{dx}{\sqrt{x \log(1/x)}}$ .
- d) Prove that  $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$ .
- e) Evaluate  $\int_0^{\infty} e^{-4x} x^{\frac{1}{2}} dx$ .

