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Sl. No.

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VI Semester B.Sc. Examination, September/October - 2022
(Semester Scheme) (CBCS)
MATHEMATICS
Algebra - IV and Complex Analysis - I (DSE)

Time : 3 Hours **Max. Marks : 80**

Instructions : *1) Answer all the five full questions.*

2) First question carries 20 marks and remaining questions carry 15 marks.

1. Answer any Ten questions. Each question carries Two marks.

a) In a vector space V over F, prove that $C(-\alpha) = (-C)\alpha = -(C\alpha)\ \forall\ C \in F$, $\alpha \in V$.

b) If S and T are any two subsets of a vector space V then show that $S \subset T \Rightarrow L(S) \subset L(T)$ where L(S) is the linear span of S.

c) Give example for :

i) A finite dimensional vector space.

ii) An infinite dimensional vector space.

d) Find the inverse of the matrix $\begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$ using linear transformation.

e) Find the matrix of the linear transformation $T : R^2 \rightarrow R^3$ defined by $T(x, y) = (x + y, x, 3x - y)$ w.r.t. standard basis.

P.T.O.

f) Let $T_1 : R^3 \to R^2$ and $T_2 : R^3 \to R^2$ defined by $T_1(x, y, z) = (y, x + z)$ and $T_2(x, y, z) = (2z, x - y)$. Find $2T_1 + T_2$.

g) Express $2x + y = 5$ interms of conjugate co-ordinates.

h) Evaluate : $\lim_{z \to i} \frac{z^2 + 1}{z^6 + 1}$

i) Show that $u = e^x \sin y$ is harmonic.

j) Write the transformation which gives reflection and translation of z.

k) Find the cross ratio of 1, – 1, i, –i.

l) Find the bilinear transformation $f(z) = \frac{az + b}{cz + d}$ whose fixed points are 0 and 1 by taking c = d = 1.

2. Answer any Three questions. Each question carries Five marks.

a) Let V={(x, y)|x, y ∈ ℝ}. Show that V is a vector space over the set of real numbers ℝ.

b) Verify the subset $S = \left\{(x_1, x_2, x_3) \mid x_1^2 + x_2^2 + x_3^2 \leq 1\right\}$ is a subspace or not of $V_3(R)$.

c) If V_1, V_2, V_3 are linearly independent in $V_3(R)$ then show that $V_1 + V_2$, $V_2 + V_3$, $V_3 + V_1$ are also linearly independent in $V_3(R)$.

d) If a vector space V is of finite dimension n then show that :

i) any set of (n + 1) vectors of V will be linearly dependent.

ii) no set of (n – 1) vectors of V can span V.

e) Find the co-ordinates of (–3, 1, 0) relative to the basis (1, 1, 1), (1, 2, 3) and (1, 0, 0).

3. Answer any Three questions. Each question carries Five marks.

   a) Verify whether the transformation $T : R^2 \rightarrow R^3$ defined by $T(x, y) = (x + y, x - y, y)$ is linear or not.

   b) Prove that the transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x, y, -z)$ is an automorphism. Find its order.

   c) Find the rank, range space, nullity and null space of the transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x - y, y + z, x + z)$. Further verify Rank - Nullity theorem.

   d) Find the eigenvalues and eigen vectors of the linear transformation $T : R^2 \rightarrow R^2$ defind by $T(x, y) = (x + 2y, 2x - y)$.

   e) Show that the transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + z, x - z, y)$ is invertible and find T^{-1}.

4. Answer any Three questions. Each question carries Five marks.

   a) Find the equation of a circle described on the line joining the points $1 + 2i$, $5 - 6i$ as ends of the diameter. Further find centre and radius.

   b) Find whether the points (2, 1), (3, 5), (–2, 0) and (1, –1) are concyclic or not.

   c) If $f(z) = \sin z$, find $f'(z)$ at $z = i$ using the definition of derivative.

   d) Verify the function $W = e^{\bar{z}}$ is analytic or not.

   e) Find the analytic function whose real part is $u = \frac{y}{x^2 + y^2}$. Also find the imaginary part.

5. Answer any Three questions. Each question carries Five marks.

   a) Discuss the transformation $W = Z^2$.

   b) Find the image of upper half z-plane under the transformation $W = \frac{i(z - i)}{z + i}$.

c) Show that every bilinear transformation is a combination of transformations like translation, inversion, rotation and magnification.

d) Under What condition $|z|=1$ is mapped to a straight line under bilinear transformation $W = \frac{az+b}{cz+d}, ad - bc \neq 0$.

e) Find the bilinear transformation which maps 0, 1, ∞ to $-5, -1, 3$.

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