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MA-5265

Sl. No.

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VI Semester B.Sc. Examination, September/October - 2022

(Semester Scheme) (CBCS)

MATHEMATICS

Algebra - IV and Complex Analysis - I (DSE)

Time : 3 Hours

Max. Marks : 80

Instructions : 1) Answer all the five full questions.

2) First question carries 20 marks and remaining questions carry 15 marks.

1. Answer any Ten questions. Each question carries Two marks.

a) In a vector space V over F , prove that $C(-\alpha) = (-C) \alpha = -(C\alpha) \forall C \in F, \alpha \in V$.

b) If S and T are any two subsets of a vector space V then show that $S \subset T \Rightarrow L(S) \subset L(T)$ where $L(S)$ is the linear span of S .

c) Give example for :

i) A finite dimensional vector space.

ii) An infinite dimensional vector space.

d) Find the inverse of the matrix $\begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$ using linear transformation.

e) Find the matrix of the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x, y) = (x + y, x, 3x - y)$ w.r.t. standard basis.

P.T.O.

Ⓣ Let $T_1 : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ and $T_2 : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T_1(x, y, z) = (y, x + z)$ and $T_2(x, y, z) = (2z, x - y)$. Find $2T_1 + T_2$.

g) Express $2x + y = 5$ in terms of conjugate co-ordinates.

h) Evaluate: $\lim_{z \rightarrow i} \frac{z^2 + 1}{z^6 + 1}$

i) Show that $u = e^x \sin y$ is harmonic.

j) Write the transformation which gives reflection and translation of z .

k) Find the cross ratio of $1, -1, i, -i$.

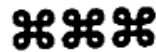
l) Find the bilinear transformation $f(z) = \frac{az + b}{cz + d}$ whose fixed points are 0 and 1 by taking $c = d = 1$.

2. Answer any Three questions. Each question carries Five marks.

- a) Let $V = \{(x, y) | x, y \in \mathbb{R}\}$. Show that V is a vector space over the set of real numbers \mathbb{R} .
- b) Verify the subset $S = \{(x_1, x_2, x_3) | x_1^2 + x_2^2 + x_3^2 \leq 1\}$ is a subspace or not of $V_3(\mathbb{R})$.
- c) If V_1, V_2, V_3 are linearly independent in $V_3(\mathbb{R})$ then show that $V_1 + V_2, V_2 + V_3, V_3 + V_1$ are also linearly independent in $V_3(\mathbb{R})$.
- d) If a vector space V is of finite dimension n then show that :
 i) any set of $(n + 1)$ vectors of V will be linearly dependent.
 ii) no set of $(n - 1)$ vectors of V can span V .
- e) Find the co-ordinates of $(-3, 1, 0)$ relative to the basis $(1, 1, 1), (1, 2, 3)$ and $(1, 0, 0)$.

3. Answer any Three questions. Each question carries Five marks.
- Verify whether the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x, y) = (x + y, x - y, y)$ is linear or not.
 - Prove that the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x, y, -z)$ is an automorphism. Find its order.
 - Find the rank, range space, nullity and null space of the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, y + z, x + z)$. Further verify Rank - Nullity theorem.
 - Find the eigenvalues and eigen vectors of the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (x + 2y, 2x - y)$.
 - Show that the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x + z, x - z, y)$ is invertible and find T^{-1} . <https://www.uomonline.com>
4. Answer any Three questions. Each question carries Five marks.
- Find the equation of a circle described on the line joining the points $1 + 2i, 5 - 6i$ as ends of the diameter. Further find centre and radius.
 - Find whether the points $(2, 1), (3, 5), (-2, 0)$ and $(1, -1)$ are concyclic or not.
 - If $f(z) = \sin z$, find $f'(z)$ at $z = i$ using the definition of derivative.
 - Verify the function $W = e^{\bar{z}}$ is analytic or not.
 - Find the analytic function whose real part is $u = \frac{y}{x^2 + y^2}$. Also find the imaginary part.
5. Answer any Three questions. Each question carries Five marks.
- Discuss the transformation $W = Z^2$.
 - Find the image of upper half z -plane under the transformation $W = \frac{i(z - i)}{z + i}$.

- c) Show that every bilinear transformation is a combination of transformations like translation, inversion, rotation and magnification.
- d) Under What condition $|z|=1$ is mapped to a straight line under bilinear transformation $W = \frac{az + b}{cz + d}, ad - bc \neq 0$.
- e) Find the bilinear transformation which maps $0, 1, \infty$ to $-5, -1, 3$.



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