93705 **M-795**

Sl. No.

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V Semester III B.Sc. Examination, March/April - 2021
(Semester Scheme) (CBCS)
MATHEMATICS
DSE : Real Analysis - II and Algebra - III

Time : 3 Hours **Max. Marks : 80**

Instructions : *1) Answer all the five full questions.*

2) First question carries 20 marks and remaining questions carry 15 marks.

1. **Answer any Ten questions. Each question carries two marks.**

a) Find infimum and supremum of the set $\left\{\frac{1}{2},\frac{3}{4},\frac{7}{8},\frac{15}{16},........\right\}$.

b) Find the limit of the sequence $\left\{\left(1+\frac{3}{n}\right)^{n+1}\right\}$.

c) Show that the sequence $\left\{\frac{2n+3}{3n+5}\right\}$ is monotonically increasing.

d) Find the infinite series whose n^{th} partial sum is $\frac{n}{n+1}$.

e) Examine the convergence of the series $\sum_{n=1}^{\infty}\cos\left(\frac{1}{n}\right)$.

f) If the series $\sum_{n=1}^{\infty}a_n$ converges, then prove that $\lim_{n\to\infty}a_n=0$.

g) Define an integral domain. Give an example of a ring which is not an integral domain.

P.T.O.

h) Find all the units and zero divisions of the commutative ring Z_{12}.

i) Show that $1-\sqrt{5}$ and $3+\sqrt{5}$ are associates in $Z\left(\sqrt{5}\right)$.

j) If ϕ is a homomorphism from a ring R into a ring R' then prove that $\phi(-a) = -\phi(a)\ \forall\ a \in R$.

k) Test the reducibility of $x^2 + 1$ over Z_5.

l) Define prime ideal and maximal ideal of a commutative ring.

2. **Answer any three questions. Each question carries five marks.**

a) Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is monotonically increasing and bounded above and hence show that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

b) Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\dots\dots\dots+\frac{1}{n!}$ is convergent.

c) Show that the sequence $\{x_n\}$ given by $x_1 = \sqrt{3}$ and $x_{n+1} = \sqrt{3x_n}$ converges to 3.

d) Prove that a convergent sequence is bounded. Does the converse true? Justify.

e) Discuss the convergence of the sequences

i) $\left\{\frac{(n+3)(1+2+\dots..+n)}{1^2+2^2+\dots\dots+n^2}\right\}$

ii) $\left\{\frac{n(\log(n+1)-\log n)}{\sin\left(1/n\right)}\right\}$

3. **Answer any three questions. Each question carries five marks.**

a) State and prove Raabe's test for the convergence of a series of positive terms.

b) Test the convergence of the series $1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+..........$

c) Test the convergence of $\sum_{n=1}^{\infty}\left(\frac{nx}{n+1}\right)^n$.

d) Discuss the convergence of $\frac{1}{1.3}-\frac{1}{2.4}+\frac{1}{3.5}-\frac{1}{4.6}+..........$

e) Sum to infinity the series $1+\frac{4}{6}+\frac{4.5}{6.9}+\frac{4.5.6}{6.9.12}+..........$

4. **Answer any three questions. Each question carries five marks.**

a) Show that the set $Z\left[\sqrt{2}\right]=\left\{a+b\sqrt{2}\ /\ a,b\in Z\right\}$ is a commutative ring with unity.

b) Prove that a commutative ring Z_n is an integral domain if and only if n is a prime number.

c) Show that the intersection of any two subrings of a ring R is again a subring of R.

d) Show that the set of polynomials $f(x)$ with $f(3) = f(7) = 0$ is an ideal.

e) If $\alpha=a+b\sqrt{7}\in Z(\sqrt{7})$ and $N(\alpha) = a^2 - 7b^2$ prove that $N(\alpha\beta) = N(\alpha)N(\beta)\ \forall\ \alpha, \beta \in Z(\sqrt{7})$ and α is a unit in $Z(\sqrt{7})$ if and only if $N(\alpha) = \pm 1$.

5. **Answer any three questions. Each question carries five marks.**

a) Show that the polynomial $x^3 - 9$ is reducible over the field Z_{11} and find the factors.

b) Find the $g\ c\ d$ of $f(x) = x^3 - 3x - 2$ and $g(x) = x^5 - x^4 - 6x^3 - 2x^2 + 5x + 3$ over Q and express it as a linear combination of $f(x)$ and $g(x)$.

c) Let f be a homomorphism from a ring R into a ring R' with Kernel k then prove that f(R) is isomorphic to a quotient ring R/K.

d) Prove that the Kernel of homomorphism $f : R \rightarrow R'$ is an ideal of R.

e) Using Eisenstein's criterion, show that the polynomial $f(x) = 8x^3 + 6x^2 - 9x + 24$ is irreducible over Q.

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