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M-795

Sl. No.

Total No. of Pages : 4

V Semester III B.Sc. Examination, March/April - 2021
(Semester Scheme) (CBCS)
MATHEMATICS
DSE : Real Analysis - II and Algebra - III

Time : 3 Hours

Max. Marks : 80

- Instructions :** 1) *Answer all the five full questions.*
2) *First question carries 20 marks and remaining questions carry 15 marks.*

1. **Answer any Ten questions. Each question carries two marks.**

- a) Find infimum and supremum of the set $\left\{ \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \dots \right\}$.
- b) Find the limit of the sequence $\left\{ \left(1 + \frac{3}{n} \right)^{n+1} \right\}$.
- c) Show that the sequence $\left\{ \frac{2n+3}{3n+5} \right\}$ is monotonically increasing.
- d) Find the infinite series whose n^{th} partial sum is $\frac{n}{n+1}$.
- e) Examine the convergence of the series $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$.
- f) If the series $\sum_{n=1}^{\infty} a_n$ converges, then prove that $\lim_{n \rightarrow \infty} a_n = 0$.
- g) Define an integral domain. Give an example of a ring which is not an integral domain.

P.T.O.

- h) Find all the units and zero divisions of the commutative ring Z_{12} .
- i) Show that $1 - \sqrt{5}$ and $3 + \sqrt{5}$ are associates in $Z(\sqrt{5})$.
- j) If ϕ is a homomorphism from a ring R into a ring R' then prove that $\phi(-a) = -\phi(a) \forall a \in R$.
- k) Test the reducibility of $x^2 + 1$ over Z_5 .
- l) Define prime ideal and maximal ideal of a commutative ring.

2. Answer any three questions. Each question carries five marks.

- a) Show that the sequence $\left\{ \left(1 + \frac{1}{n} \right)^n \right\}$ is monotonically increasing and

bounded above and hence show that $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$.

- b) Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$ is convergent.
- c) Show that the sequence $\{x_n\}$ given by $x_1 = \sqrt{3}$ and $x_{n+1} = \sqrt{3x_n}$ converges to 3.
- d) Prove that a convergent sequence is bounded. Does the converse true? Justify.
- e) Discuss the convergence of the sequences

- i) $\left\{ \frac{(n+3)(1+2+\dots+n)}{1^2+2^2+\dots+n^2} \right\}$

- ii) $\left\{ \frac{n(\log(n+1) - \log n)}{\sin(1/n)} \right\}$

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3. Answer any three questions. Each question carries five marks.

- a) State and prove Raabe's test for the convergence of a series of positive terms.
- b) Test the convergence of the series $1 + \frac{1}{2}x + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + \dots$
- c) Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{nx}{n+1}\right)^n$.
- d) Discuss the convergence of $\frac{1}{1.3} - \frac{1}{2.4} + \frac{1}{3.5} - \frac{1}{4.6} + \dots$
- e) Sum to infinity the series $1 + \frac{4}{6} + \frac{4.5}{6.9} + \frac{4.5.6}{6.9.12} + \dots$

4. Answer any three questions. Each question carries five marks.

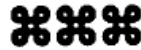
- a) Show that the set $Z[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in Z\}$ is a commutative ring with unity.
- b) Prove that a commutative ring Z_n is an integral domain if and only if n is a prime number.
- c) Show that the intersection of any two subrings of a ring R is again a subring of R .
- d) Show that the set of polynomials $f(x)$ with $f(3) = f(7) = 0$ is an ideal.
- e) If $\alpha = a + b\sqrt{7} \in Z(\sqrt{7})$ and $N(\alpha) = a^2 - 7b^2$ prove that $N(\alpha\beta) = N(\alpha)N(\beta) \forall \alpha, \beta \in Z(\sqrt{7})$ and α is a unit in $Z(\sqrt{7})$ if and only if $N(\alpha) = \pm 1$.

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5. Answer any three questions. Each question carries five marks.

- a) Show that the polynomial $x^3 - 9$ is reducible over the field Z_{11} and find the factors.
- b) Find the g c d of $f(x) = x^3 - 3x - 2$ and $g(x) = x^5 - x^4 - 6x^3 - 2x^2 + 5x + 3$ over Q and express it as a linear combination of $f(x)$ and $g(x)$.
- c) Let f be a homomorphism from a ring R into a ring R' with Kernel k then prove that $f(R)$ is isomorphic to a quotient ring R/K .
- d) Prove that the Kernel of homomorphism $f: R \rightarrow R'$ is an ideal of R .
- e) Using Eisenstein's criterion, show that the polynomial $f(x) = 8x^3 + 6x^2 - 9x + 24$ is irreducible over Q .



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