20762 **M-8403**

Sl.No. Total No. of Pages : 3

V Semester B.Sc. Examination, Oct./Nov. - 2019
(Semester Scheme) (2015-16 Batch on Wards)
MATHEMATICS
Algebra - III and Calculus - II (Paper - VI)

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer all the sections.

SECTION - A

I. Answer any Eight questions. Each question carries two marks.

a) In an Integral Domain D if a.b = a.c, a ≠ 0 then prove that b = c.

b) Show that the set of matrices of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ is a subring of the ring of 2×2 matrices with integral elements.

c) Define principal ideal and maximal ideal.

d) Verify whether f : $\mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x \forall x \in R$ is a homomorphism or not

e) Find all the rings Z_n such that $x^5 - 10x + 12$ is divisible by $x^2 + 2$.

f) Find all the zeroes of $x^4 + 3x^3 + 2x + 4$ in $Z_3 [x]$.

g) If $f(x) = x+1, \forall x \in [0,1]$ and $P = \{0, 1/5, 1/3, 1\}$ Find $L(P,f)$ and $U(p,f)$

h) If f is bounded and k < 0, then prove that $\overline{\int_a^b} kf(x)dx = k\underline{\int_a^b} f(x)dx$

i) Define Lower and upper Riemann integral of a bounded function $f(x)$ over [a,b].

j) Verify whether the function

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational is} \end{cases}$$

R-integrable over [a,b] or not

k) State first mean value theorem of Integral calculus.

l) If f is a bounded function and P is a partition of [a,b] then show that $U(p,f) \geq L(p,f)$

P.T.O.

SECTION - B

II. Answer any eight questions. Each question carries four marks.

a) Show that the set $z_6 = \{0,1,2,3,4,5\}$ is a commutative ring with unity w.r.t. addition modulo 6 and multiplication modulo 6

b) Prove that every field is an integral domain

c) Prove that every integral Domain can be embedded isomorphically into the field of Quotients.

d) Show that intersection of any two ideals of a ring R is also an ideal of R.

e) If $\alpha = a + b\sqrt{7} \in \mathbb{Z}[\sqrt{7}]$ and $N(\alpha) = a^2 - 7b^2$, then prove that $N(\alpha\beta) = N(\alpha)N(\beta)$ for any two elements $\alpha, \beta \in \mathbb{Z}[\sqrt{7}]$

f) If k is an ideal of a commutative ring R prove that the transformation $f: R \to R/K$ defined by $f(a) = k + a$ is an homomorphism with K as its kernel.

g) Find the G.C.D of the polynomials $f(x) = x^3 + 1$ and $g(x) = x^3 + 2x^2 - 2x + 3$ and express it as a linear combination of $f(x)$ and $g(x)$

h) Show that $2 - \sqrt{17}$ and $-9 - 2\sqrt{17}$ are associates in the domain $\mathbb{Z}[\sqrt{17}]$

i) Test the equation $6x^4 - 7x^3 + 6x^2 - 1 = 0$ for rational roots.

j) Show that $x^3 - 9$ is reducible over $\mathbb{Z}_{11}$ and find its factors.

SECTION - C

III. Answer any eight questions. Each question carries four marks.

a) If $f: [a, b] \to R$ is R-integrable then show that $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$ where m is infimum and M is supremum of $f(x)$ on [a,b].

b) State and prove Darboux's theorem.

c) Show that every continous function is R-integrable

d) Show that $f(x) = x^2$ is R-integrable over [0,1]

e) If f is R-integrable then show that $|f|$ is also R-integrable and show that the converse is not true in general

f) If f and g are R-integrable then show that f + g is also R-integrable.

g) Show that $f(x) = 2-3x$ is R-integrable over [1,3] and prove that $\int_0^1 (2-3x)dx = \frac{1}{2}$

h) If $f(x) = \frac{1}{a^n}$ *for* $\frac{1}{a^{n+1}} < x \le \frac{1}{a^n}$, $n = 0,1,2,... = 0$ for $x = 0$ Show that $f(x)$ is R-integrable over [0,1] and show that $\int_0^1 f(x)dx = \frac{a}{1+a}$

i) If $f(x)$ is R-integrable over [a,b] where a < c < b then prove that $\int_a^b f\,dx = \int_a^c f dx + \int_c^b f dx$

j) State and prove fundamental theorem of Integral calculus.

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