93704 MA-5264

Sl. No.

Total No. of Pages : 4

IV Semester B.Sc. Examination, September/October - 2022
(Semester Scheme) (CBCS)
MATHEMATICS (Paper - IV)
Differential Equations - II and Real Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instructions :
1) *Answer all the five questions.*
2) *First question carries 20 marks and remaining questions carries 15 marks.*

1. Answer any Ten questions. Each Question carries two marks.

a) Show that the equation :

$$\sin x\frac{d^2y}{dx^2}-\cos x\frac{dy}{dx}+2(\sin x)y=0 \text{ is Exact}$$

b) Test for integrability of the equation

$$(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$$

c) Solve : $\frac{dx}{y^2}=\frac{dy}{x^2}=\frac{dz}{x^2y^2z^2}$

d) Form a partial differential equation by eliminating the arbitrary constants a and b from $Z=(x^2+a)(y^2+b)$

e) Solve : $\sqrt{p}+\sqrt{q}=1$

f) Write Charpit's auxillary equation for $f(x, y, z, p, q)=0$.

g) If $f(x)=x+1$ $\forall x\in[0,1]$ and $P=\left\{0,\frac{1}{3},\frac{2}{3},1\right\}$ is a partition of [0,1] find $L(p,f)$ and $U(p,f)$.

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h) Give an example of a bounded function which is not Riemann integrable.

i) State second mean value theorem of integral calculus.

j) Evaluate $\int_C (2x+y)\,dx + (3y+x)\,dy$ where C is the line joining the points (0, 1) and (2, 5).

k) Evaluate $\int_0^a \int_0^b xy(x-y)\,dx\,dy$

l) Evaluate $\int_0^1 \int_1^2 \int_1^2 x^2 yz\,dz\,dy\,dx$

2. Answer any three questions. Each question carries five marks.

a) Solve : $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = 8x^3 \sin x^2$ by changing independent variable.

b) Solve : $\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 5y = 0$ by reducing to normal form.

c) Solve : $\frac{d^2y}{dx^2} + y = \cot x$ by the method of variation of parameters.

d) Test the equation $(1+x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \sec^2 x$ for exactness and solve.

e) Solve the Simultaneous equations

$$\frac{l\,dx}{mn(y-z)} = \frac{m\,dy}{ln(z-x)} = \frac{n\,dz}{ml(x-y)}$$

3. **Answer any three questions. Each question carries five marks.**

a) Form a partial differential equation by eliminating the arbitrary function from $\phi\left(z^2 - xy, \frac{x}{z}\right) = 0$.

b) Solve the partial differential equation $p + q = \sin x + \sin y$

c) Solve : $z = pq$ by Charpit's method.

d) Solve : $p \tan x + q \tan y = \tan z$.

e) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x+3y)$

4. **Answer any three questions. Each question carries five marks.**

a) Show that $f(x) = 2x + 1$ is Riemann integrable on $[1, 2]$ and $\int_1^2 f(x)\,dx = 4$

b) Let $f : [a, b] \rightarrow R$ is bounded over [a, b] with partition P. Show that $m(b-a) \leq L(P, f) \leq U(P, f) \leq M(b-a)$ where m = inf. f and M = sup.f over [a, b]

c) Show that the function f defined on [0, 1] by $f(x) = 2rx$ if $\frac{1}{r+1} < x \leq \frac{1}{r}$, $r = 1, 2, 3, \ldots$ is R - integrable over [0, 1] and show that $\int_0^1 f(x)\,dx = \frac{\pi^2}{6}$.

d) If f and g are Riemann integrable over $[a, b]$, then prove that $f + g$ is also Riemann integrable over $[a, b]$.

e) Compute $\int_C xy\,dx + x^2 z\,dy + xyz\,dz$ with $x = e^t, y = e^{-t}, z = t^3, 0 \leq t \leq 1$.

5. **Answer any three questions. Each question carries five marks.**

a) Evaluate : $\iint_D \sin y\,dx\,dy$ where D is the region bounded by the lines $2y = x$, $y = 2x$ and $x = \pi$.

b) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\,dx$ by changing the order of integration.

c) Find the area of the surface $Z = \sqrt{x^2 + y^2}$, $\frac{1}{4} < x^2 + y^2 < 1$

d) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \int_0^{\sqrt{a^2 - x^2 - y^2}} xyz\, dz\, dy\, dx$

e) Find the volume inside the cylinder $x^2 + y^2 = 9$ above the plane $z = 0$ and below the plane $x + y = z$.