



27323(New)

B.Sc. III Semester Degree Examination, March/April - 2021

MATHEMATICS-III

Paper : 3

(New)

Time : 3 Hours

Maximum Marks : 80.

Instructions to Candidates:

- 1) PART - A All questions are compulsory.
- 2) PART - B Solve any Five questions from Seven questions.
- 3) Write the question number correctly.

PART - A

I. Answer the following questions.

(10×2=20)

1) Define Infimum and Suprimum of the sequence.

2) State P - Series.

3) State Raabe's Test.

4) Discuss the convergence of the series  $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$ .

5) Test the convergence of  $\sum \frac{1}{\sqrt{n}} \tan \frac{1}{n}$ .

6) Find the angle between the radius vector and the tangent for the curve  $r = a(1 - \cos \theta)$ .

7) For the curve  $r = a\theta$  show that  $p = \frac{r^2}{\sqrt{r^2 + a^2}}$ .

8) Find the ratio of the Polar subnormal to the polar subtangent for the curve

$$r = ae^{b\theta^2}.$$



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9) Evaluate  $\int_0^{\frac{\pi}{4}} \tan^n x dx$ .

10) Evaluate  $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x dx$ .

**PART - B**

Answer any Five complete questions.

(5×12=60)

**II.** 11) State and prove De - Alembert's ratio test.

12) Examine the convergence of the series  $1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$  by Raabes test.

**III.** 13) State Cauchy's root test and hence test the convergence of the series

$$\sum \left( \frac{n^x}{n+1} \right)^n$$

14) Sum the series by C+ is method  $\cos \alpha + \cos(\alpha + 2\beta) + \dots +$  to n term.

**IV.** 15) Show that the following pair of curves intersect orthogonally  $r = a\theta, r = \frac{a}{\theta}$ .

16) For the curve cardioid  $r = a(1 + \cos \theta)$  Show that  $2ap^2 = r^3$ .

**V.** 17) Derive the length of the perpendicular from the pole to the tangent at a point to the curve.

18) (i) Find the pedal equation of the curve  $r = ae^{\theta \cos \alpha}$

(ii) Find the angle between the radius vector and the tangent for the curve

$$r^2 = a^2 \cos 2\theta \text{ at } \theta = \frac{\pi}{6}$$



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VI. 19) Show that the curve  $r^2 = a^2 \sec 2\theta$  the length of the perpendicular from the pole to the tangent  $a\sqrt{\cos 2\theta}$ .

20) Show that the angle between the normal at any point  $(r, \theta)$  on the curve  $r^n = a^n \cos n\theta$ , and the initial line is  $(n+1)\theta$ .

VII. 21) Find the reduction formula for  $\int \sec^n x dx$ .

22) Evaluate (i)  $\int_0^1 x^{1/2} \sqrt{(1-x)^3} dx$

(ii)  $\int_0^{\infty} \frac{x}{(4+x^2)^{3/2}} dx$

VIII.23) Evaluate (i)  $\int_0^{\pi} x \sin^7 x \cos^2 x dx$

(ii)  $\int_0^{\pi/4} \tan^4 x dx = \frac{3\pi - 8}{12}$

24) Compute the definite integral  $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$ , where ' $\alpha$ ' is a parameter using Leibnitz's rule of differentiation under integral sign.