51135 **M-4063**

Sl.No. **Total No. of Pages : 3**

I Semester B.Sc. (Hon's) Data Science and Artificial Intelligence Examination, September - 2021
(Scheme : CBCS)
MATHEMATICS (Paper - I)
Basics of Mathematics and Calculus for Science

Time : 3 Hours **Max. Marks : 80**

Instructions: 1) **Answer all the five quesitons.**
2) **First question carries 20 marks and remaining questions carry 15 marks.**

1. **Answer any ten questions. Each question carries two marks. [10 × 2 = 20]**

a) Define function. Give an example for a function which is one-one but not onto. [2]

b) If $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then prove that $\frac{dy}{dx}=\frac{2}{1+x^2}$. [2]

c) Find the area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$. [2]

d) State Rolle's theorem. [2]

e) If $f(x) = (ax+b)^m$, where m>o, then find the n^{th} derivative of $f(x)$ [2]

f) State Green's theorem in plane. [2]

g) Solve the differential equation $\frac{dy}{dx}=e^{x+y}+x^2e^{y}$. [2]

h) Find $L^{-1}\left\{\frac{1}{s-a}+\frac{1}{s-b}\right\}$ where 'a' and 'b' are constants. [2]

i) Form the differential equation corresponding to y = a cos (mx+b) by eliminating a and b. [2]

j) Show that if a|b and c|d, then ac|bd. [2]

k) Find the remainder and quotient when −1023 is divided by 4. [2]

l) When do you say that a≡b (mod n)? Give an example. [2]

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2. **Answer any three questions. Each question carries five marks.[3 × 5 = 15]**

a) Find the reduction formula for $\int \tan^n x \, dx$ Hence evaluate $\int \tan^4 x \, dx$.[5]

b) i) Evaluate $\lim_{x \to 0} \frac{\sin 4x}{\sin 2x}$. [5]

ii) If $y = \frac{x^5 - \cos x}{\sin^2 x}$, then find $\frac{dy}{dx}$.

c) If $u = e^x (x \cos y - y \sin y)$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. [5]

d) Find the equation of the normal to the curve $2x^2 - y^2 = 14$ parallel to $x + 3y = 4$. [5]

e) Show that $f(x,y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} \text{ if } (x,y) \neq (0,0) \\ 0 \quad \text{ if } (x,y) = (0,0) \end{cases}$ is not continuous at (0,0).[5]

3. **Answer any three questions. Each question carries five marks.[3 × 5 = 15]**

a) Find the n[th] derivative of $f(x) = \frac{1}{6x^2 - 5x + 1}$ at $x = 0$. [5]

b) State mean value theorem, and verify the same for $f(x) = x + \frac{1}{x}$ on the interval $\left[\frac{1}{2}, 2\right]$. [5]

c) Sketch the region of integration and evaluate the double integral $\int_0^{\pi} \int_0^{\sin x} x \, dy \, dx$. [5]

d) When do you say that a function $f(x)$ is decreasing on [a,b]? Find an interval on which the function $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + 4x + 5$ is decreasing. Justify your answer. [5]

e) Find the curl and divergence of the function $\vec{F}$, where $\vec{F}$ is:

$$\vec{F} = (2yz \quad x)\hat{i} + (2xz \quad y)\hat{j} + (2xy \quad z)\hat{k}.$$ [5]

4. Answer any three questions. Each question carries five marks.[3 × 5 = 15]

a) i) Find the order and degree of the differential equation. [5]

$$\left(1+\left(\frac{dy}{dx}\right)^{2}\right)^{\frac{3}{2}} = a\left(\frac{d^2y}{dx^2}\right)$$

ii) Solve $\cos x\dfrac{dy}{dx} + y\sin x = 1$

b) Obtain the partial differential equation of all spheres whose centers lie on the plane z=0 and whose radius is constant and equal to 'r'. [5]

c) Evaluate i) $L\{\sin 3t \cos 2t\}$. [5]

ii) $L^{-1}\left\{\log\left(\dfrac{s+1}{s+2}\right)\right\}$.

d) Solve: $\left(\dfrac{x+y-1}{x+y-2}\right)\dfrac{dy}{dx} = \dfrac{x+y+1}{x+y+2}$. [5]

e) Solve: $(D^2 - 3D + 2)y = 6e^{-3x} + \sin 2x$, where $D=\dfrac{d}{dx}$. [5]

5. Answer any three questions. Each question carries five marks.[3 × 5 = 15]

a) Define Greatest common divisor (G.C.D) of two integers 'a' and 'b'. Express the GCD of 306 and 657 as a linear sum of those two numbers.[5]

b) Let G.C.D of 'a' and 'b' be 1. If a|c and b|c, then show that ab|c and hence derive a divisibility rule for 72.

c) State fundamental theorem of Arithmetic. Find the prime factorizations of 22680 and 23814. Find also their least common multiple (LCM) through their prime factorization. [5]

d) State wilson's theorem. Use wilson's theorem to find the remainder when 21! is divided by 23. [5]

e) Find the remainder when 5^{92} is divided by 7. [5]

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